

Il problema dei compleanni

- Note

- Autore

Claudio Marsan
Liceo Cantonale di Mendrisio
Via Agostino Maspoli
CH-6850 Mendrisio (Switzerland)
e-mail: claudio.marsan@liceomendrisio.ch

- Versione

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- La formula di Stirling

```
[ > restart;  
[ > S := n -> sqrt(2 * Pi * n) * n^n * exp(-n);
```

$$S := n \rightarrow \sqrt{2 \pi n} n^n e^{(-n)}$$

Alcuni esempi:

```
[ > evalf(S(1));
```

.9221370087

```
[ > evalf(S(5));
```

118.0191679

```
[ > evalf(S(50));
```

.3036344594 10⁶⁵

Vale:

```
[ > Limit(S(n)/n!, n = +infinity) = limit(S(n)/n!, n =  
+infinity);
```

$$\lim_{n \rightarrow \infty} \frac{\sqrt{2} \sqrt{\pi n} n^n e^{(-n)}}{n!} = 1$$

e dunque S(n) e n! sono asintoticamente identiche.

Confrontiamo alcuni valori:

```
[ > for n from 1 to 15 do printf(`%2.0f %15.0f %16.2f  
%1.5f\n`, n, n!, evalf(S(n)), evalf(S(n)/n!)); od;
```

1	1	.92	.92213
2	2	1.91	.95950
3	6	5.83	.97270
4	24	23.50	.97942
5	120	118.01	.98349
6	720	710.07	.98621
7	5040	4980.39	.98817
8	40320	39902.39	.98964

```

9          362880          359536.87   .99078
10         3628800         3598695.62  .99170
11        39916800         39615625.07  .99245
12       479001600         475687486.70  .99308
13      6227020800         6187239477.00  .99361
14     87178291200         86661001750.00  .99406
15    1307674368000       1300430722000.00  .99446

```

```
[ >
```

- Il problema dei compleanni

Qual è la probabilità che fra n persone ce ne siano almeno due che compiono gli anni nello stesso giorno?

```
[ > restart;
```

```
[ > casi_possibili := n -> 365^n;
```

$$\text{casi_possibili} := n \rightarrow 365^n$$

```
[ > casi_non_favorevoli := n -> 365!/(365-n)!;
```

```

casi_non_favorevoli := n -> 2510412867555873229292944374881202770516552026987\
607976687259519390110613822093741966601800900025416937617231436098232866\
070807112336997985344536791065387238359970435553274093767809149142944086\
431604692507451013484702554601409800590796554104119549610531188617337343\
514551719328276084775588229169021353912347918627470151939680850494072260\
703300124632839880055048742799987669041697343786107818534466796687151104\
965388813013683619901052918005612584454948864861768291582634756414899098\
413806780999960468748814673483734069935983879112499595758453887361666153\
309325355125684505604638873812970295138115186141368892298651000544094394\
301469924411255575527914076049276425374025041039105642197900328960000000\
00000000000000000000000000000000000000000000000000000000000000000000\
0000000000  $\frac{1}{(365 - n)!}$ 

```

```
[ > p := n -> evalf(1 -
casi_non_favorevoli(n)/casi_possibili(n));
```

$$p := n \rightarrow \text{evalf}\left(1 - \frac{\text{casi_non_favorevoli}(n)}{\text{casi_possibili}(n)}\right)$$

Alcuni esempi:

```
[ > p(2);
```

```
.002739726027
```

```
[ > p(10);
```

```
.1169481777
```

```
[ > p(20);
```

```
.4114383836
```

```
[ > p(25);
```

```
.5686997040
```

```
> p(50);  
                                .9703735796  
> p(100);  
                                .9999996928
```

Tabella per i primi casi:

```
> for n from 2 to 100 do printf(`%3.0f  %1.10f\n`, n, p(n));  
od;  
2  .0027397260  
3  .0082041659  
4  .0163559125  
5  .0271355737  
6  .0404624837  
7  .0562357031  
8  .0743352924  
9  .0946238339  
10 .1169481777  
11 .1411413783  
12 .1670247888  
13 .1944102752  
14 .2231025120  
15 .2529013198  
16 .2836040053  
17 .3150076653  
18 .3469114179  
19 .3791185260  
20 .4114383836  
21 .4436883352  
22 .4756953077  
23 .5072972343  
24 .5383442579  
25 .5686997040  
26 .5982408201  
27 .6268592823  
28 .6544614723  
29 .6809685375  
30 .7063162427  
31 .7304546337  
32 .7533475278  
33 .7749718542  
34 .7953168646  
35 .8143832389  
36 .8321821064  
37 .8487340082  
38 .8640678211  
39 .8782196644  
40 .8912318098  
41 .9031516115  
42 .9140304716  
43 .9239228557
```

44 .9328853686
45 .9409758995
46 .9482528434
47 .9547744028
48 .9605979729
49 .9657796093
50 .9703735796
51 .9744319933
52 .9780045093
53 .9811381135
54 .9838769628
55 .9862622888
56 .9883323549
57 .9901224593
58 .9916649794
59 .9929894484
60 .9941226609
61 .9950887988
62 .9959095749
63 .9966043868
64 .9971904790
65 .9976831073
66 .9980957046
67 .9984400430
68 .9987263913
69 .9989636663
70 .9991595760
71 .9993207532
72 .9994528806
73 .9995608056
74 .9996486444
75 .9997198782
76 .9997774375
77 .9998237792
78 .9998609546
79 .9998906684
80 .9999143319
81 .9999331085
82 .9999479529
83 .9999596457
84 .9999688221
85 .9999759973
86 .9999815870
87 .9999859254
88 .9999892802
89 .9999918647
90 .9999938484
91 .9999953652
92 .9999965207
93 .9999973977

94 .9999980607
95 .9999985602
96 .9999989349
97 .9999992151
98 .9999994237
99 .9999995784
100 .9999996928

[La risposta al problema posto è: sono necessarie almeno 23 persone